

LITHIUM ION WAVE FUNCTION (${}_3\text{Li}^{2+}$) IN A MOMENTUM CHAMBER AT $n \leq 3$

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ABSTRACT

Lithium ion (${}_3\text{Li}^{2+}$) is a light alkali metal which is reactive and flammable when exposed to water or air. Lithium becomes ionic when lithium metal loses two of its three valence electrons. The wave function is a complex quantity that shows the characteristics of the de Broglie wave. The wave function in question is the radial wave function in the particle, because to find the probability that the electron is in the particle, only the radial part of the wave function is needed. However, there is no literature that discusses the wave function in the momentum space in the case of hydrogenic ions. Therefore, this research will examine the wave function of Lithium ion (${}_3\text{Li}^{2+}$) in the momentum space. This type of research in the preparation of this article uses non-experimental research. Based on the results of the research that has been done, the wave function of Lithium ion (${}_3\text{Li}^{2+}$) in the momentum space at $n = 3$ is obtained. The wave function consists of a radial, angular (polar and azimuth) wave function, as well as the wave function of Lithium ion (${}_3\text{Li}^{2+}$) as a whole. The fundamental difference between the wave function of Lithium ion (${}_3\text{Li}^{2+}$) in the momentum space and the wave function of hydrogenic atoms in the other momentum space is the value of the radial function caused by the difference in the value of (z) or atomic number.

Keywords: *Lithium ion, Wave function, Quantum, Momentum space*

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INTRODUCTION

Hydrogen has three isotopic forms consisting of protium, deuterium and tritium. Protium is an isotope of hydrogen that has 1 electron and 1 proton. Deuterium is an isotope of hydrogen that has 1 electron, 1 proton and 1 neutron while tritium has 1 electron, 1 proton and 2 neutrons. All three isotope forms have a single electron and have a property called hydrogenic properties. In addition to these three isotopes, there are other atoms that have the possibility of electron 1 due to the ionization process which can later qualify as hydrogenic atoms. These atoms are Beryllium, Helium, and Lithium. Of the three atoms, lithium becomes an ion when lithium metal loses two of its three electrons. This causes Lithium ions to become single-electron and hydrogenic Lithium ions so that the properties possessed by Lithium ions are the same as Hydrogen atoms (Karomah et al., 2021)

Lithium is the metal with the highest oxidation potential among the other basic elements. The lithium ion (${}_3\text{Li}^{2+}$) is a light alkali metal that is reactive and burns when exposed to water or air. In addition, lithium is also very easy to react with water to form lithium hydroxide and hydrogen gas. In everyday life, lithium is widely used as a battery for various electronic devices. Lithium batteries can be divided into two types, namely primary batteries (non-rechargeable) and secondary batteries (rechargeable) (Wijayanto et al., 2020).

The wave function (ψ) is a complex quantity that exhibits the characteristics of particle waves. The main difference between classical mechanics and quantum mechanics lies in the way they are described. A particle in classical mechanics is determined by its initial position, initial momentum, and the forces acting on it. Particles in quantum mechanics are defined by wave functions, which are expressed as solutions of Schrodinger's equations. This wave function provides information about the state of the particle, including its position, momentum and energy (Kharismawati & Supriadi, 2021).

In solving the Schrodinger Equation on ions with single electrons can be divided into two equations, namely depending on the radius and depending on the angle (Hermanto, 2016). The Schrodinger equation has several approaches, for that in obtaining the particle wave function, there are 3 conditions that must be met, including: 1) the law of conservation of energy must not be violated, 2) the de Broglie hypothesis must be obeyed, 3) the results of the equation must behave well (Damayanti et al., 2020). The Schrodinger equation in spherical coordinates can be one of the solutions to the Schrodinger equation for atoms that are hydrogenic, with the following equation.

$$\frac{\hbar^2}{2m} \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \left(\frac{\partial^2 \psi}{\partial \phi^2} \right) \right] + E \psi_{(r,\theta,\phi)} = V \psi_{(r,\theta,\phi)}$$

(Damayanti et al., 2020) (1)

The wave function of a hydrogenic atom in positional space is a mathematical description related to the possible position of the motion of electrons in an atom. In previous studies, there has been a solution to the problem of wave functions in position space with a single electron using the Schrodinger equation approach in spherical coordinates. P there is the study of Fuadah et al. (2018), which examines the solution of the Schrodinger equation of Deuterium atoms in positional space. Another study by Makmum et al. (2020), examined the function of Helium Ion waves in the representation of positional space using the Schrodinger equation. In addition, in the study of Utami et al (2019), examined the probability of the position of electrons on the tritium atom in the position space. In general, the equation of the wave function of a hydrogenic atom in positional space can be expressed as follows:

$$\psi_{n,l,m}(r, \theta, \phi) = \left[\left(\frac{2z}{na_0} \right)^3 \frac{(n-l-1)!}{(2n)(n+1)!} \right]^{\frac{1}{2}} \left[\frac{2zr}{na_0} \right]^l e^{-\left(\frac{zr}{na_0}\right)} L_{n-l-1}^{2l+1} \left[\frac{2zr}{na_0} \right] \sqrt{\frac{2l+1}{2} \frac{(l-|m|)!}{(l+|m|)!}} \frac{\sqrt{1}}{2\pi} e^{\pm im\phi} p_l^m \cos \theta$$

(Singh, 2009) (2)

In addition to being expressible in positional space, wave functions can also be expressed in momentum space using the Fourier transform. According to Podolsky & Pauling (1929), the wave function of hydrogenic atoms in momentum space can be expressed in the following equation:

$$Y_{nlm}(P, \Theta, \Phi) = \left\{ \left(\frac{2}{\pi} \frac{(n-l-1)!}{(n+l)!} \right)^{\frac{1}{2}} n^2 2^{2l+2} 2! \times \frac{n^l p^l}{(n^2 p^2 + 1)^{l+2}} C_{n-l-1}^{l+1} \left(\frac{n^2 p^2 - 1}{n^2 p^2 + 1} \right) \right\} \left\{ \frac{1}{(2\pi)^{\frac{1}{2}}} e^{\pm im\Phi} \right\} \left\{ \left(\frac{(2l+1)(l-m)!}{2(l+m)!} \right)^{\frac{1}{2}} P_l^m(\cos \Theta) \right\}$$

(Podolsky & Pauling, 1929) (3)

Based on the wave function, the fact that the problem of single-electron lithium ions can be solved using the Schrodinger equation approach, makes researchers interested in studying it further. Therefore, this study will examine the wave function of Lithium ions (${}_3\text{Li}^{2+}$) in momentum space with quantum numbers ($n \leq 3$).

METHOD

This research was carried out using non-experimental research. The form of non-experimental research used is in the form of theoretical studies by developing existing theories. This research is carried out in several steps, namely preparation, theory development, obtaining results and conclusions. The preparatory stage is the stage where literature relevant to the research is collected. The literature used as a reference in this study is in the form of books and journals as well as various relevant sources related to wave mechanics on a national and international scale. The second stage is the stage of theory development. At this stage, existing theories from various literatures, especially those related to the wave function in positional space, are developed so as to form a novelty. To express the wave function in space position on the hydrogenic atom it can be expressed in the form of spherical coordinates as follows:

$$\psi_{n,l,m}(r, \theta, \phi) = \frac{(2\gamma)^{l+1}}{(n+1)!} \left[\frac{r(n-l-1)!}{(n)(n+1)!} \right]^{\frac{1}{2}} e^{-(\gamma r)} r^l [L_{n+1}^{2l+1}(2\gamma r)] \sqrt{\frac{2l+1}{2} \frac{(l-|m|)!}{(l+|m|)!}} \frac{\sqrt{1}}{2\pi} e^{\pm im\phi} p_l^m \cos\theta \tag{4}$$

The above equation is a form of function in positional space that can be symbolized by (x, y, z, /n, l, m). To transform a function in positional space into a function in space in momentum, you can use fourier transformation

$$\psi(x) = \frac{1}{\sqrt{2\hbar\pi}} \int \varphi(p) e^{\frac{ipx}{\hbar}} dp \tag{5}$$

$$\varphi(p) = \frac{1}{\sqrt{2\hbar\pi}} \int \psi(x) e^{\frac{ipx}{\hbar}} dx \tag{6}$$

In reality, spherical coordinates are often used to describe the case of hydrogenic atoms or ions in three-dimensional space. In general (x, y, z) is used to express positional space while (p_x, p_y, p_z) used to express momentum space. In spherical coordinates it is known that the volume element is $dV = r^2 \sin \theta dr d\theta d\Phi$. If for example, it can be written as follows:

$$x = r \sin \theta \cos \Phi \qquad p_x = p \sin \theta \cos \Phi \tag{7}$$

$$y = r \sin \theta \sin \Phi \qquad p_y = p \sin \theta \sin \Phi \tag{8}$$

$$z = r \cos \theta \qquad p_z = p \cos \Phi \tag{9}$$

Where p is the magnitude of the total momentum vector, while θ and Φ are the orientations of the momentum vector relative to the cartesian coord axis. The result of the transformation of the possi wave function into momentum space can be denoted by ($p_x, p_y, p_z, /n, l, m$). The form of the function transformation is as follows:

$$(p_x, p_y, p_z, /n, l, m) = h^{-\left(\frac{3}{2}\right)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\left(\frac{2\pi f}{\hbar}\right)(xp_x + yp_y + zp_z)} (x, y, z, /n, l, m) dx dy dz \tag{10}$$

Then, if the function ($p_x, p_y, p_z, /n, l, m$) is expressed in terms of functions (p, θ , Φ) and denoted by

$$\varphi_{n,l,m}(p, \theta, \Phi) = h^{-\left(\frac{3}{2}\right)} \int_0^{\infty} \int_0^{\pi} \int_0^{2\pi} e^{-\left(\frac{2\pi f}{\hbar} rp\right)(\sin \theta \sin \theta \cos(\Phi - \phi) + \cos \theta \cos \theta)} \sqrt{\frac{1}{2\pi}} e^{\pm im\phi} p_l^m \cos \theta$$

$$\sqrt{\frac{2l+1}{2} \left(\frac{(l-|m|)!}{(l+|m|)!} \right)} \frac{(2\gamma)^{l+1}}{(n+l)!} \sqrt{\frac{\gamma(n-l-1)!}{(n+l)!}} e^{-(\gamma r)} r^l [L_{n+l}^{2l+1}(2\gamma r)] r^2 \sin \theta dr d\theta d\phi \quad (11)$$

Where $\gamma = \frac{4\pi^2 \mu e^2 z}{nh^2} = Z/na_0$, then from equation (9) will be obtained the wave function of hydrogenic atoms in momentum space which can be written as follows:

$$\varphi(p, \theta, \Phi) = \left\{ \frac{1}{(2\pi)^{\frac{1}{2}}} e^{\pm im\Phi} \sqrt{\frac{2l+1}{2} \left(\frac{(l-|m|)!}{(l+|m|)!} \right)} p_l^m \cos \theta \right\} \left\{ \frac{\pi(l)^2}{\left(\frac{z}{na_0}\right)^{\frac{3}{2}} (\hbar)^{\frac{3}{2}}} 2^{2l+4} l! \left(\frac{n(n-l-1)!}{(n+l)!} \right)^{\frac{1}{2}} \frac{\zeta^l}{(\zeta^2+1)^{l+2}} C_{n-l-1}^{l+1} \left(\frac{\zeta^2-1}{\zeta^2+1} \right) \right\} \quad (12)$$

Based on the equation above, it can be seen that there are two wave functions, namely the angular wave function and the radial wave function, where for hydrogen atoms the applicable angular wave function is:

$$Y_{lm}(\Theta, \Phi) = \left\{ \frac{1}{(2\pi)^{\frac{1}{2}}} e^{\pm im\Phi} \right\} \left\{ \left(\frac{(2l+1)(l-m)!}{2(l+m)!} \right)^{\frac{1}{2}} P_l^m(\cos \Theta) \right\}$$

While the radial wave function can be written as follows:

$$F_{n,l}(p) = \left(\frac{2(n-l-1)!}{\pi(n+l)!} \right)^{\frac{1}{2}} n^2 2^{2l+2} 2! \times \frac{n^l p^l}{(n^2 p^2 + 1)^{l+2}} C_{n-l-1}^{l+1} \left(\frac{n^2 p^2 - 1}{n^2 p^2 + 1} \right)$$

Function above is a radial wave function that applies to Hydrogen atoms where the function above is a form of function that has been expressed in atomic units for momentum or has been divided by p_0 (Bransden, 1983).

If the value of $\zeta = \frac{2\pi p}{\hbar} = \frac{np}{zp_0}$, where p_0 is the momentum of the electron in the Bohr orbit whose value is $p_0 = \frac{2\pi\mu e^2}{h}$. Based on this value, the radial momentum equation that has been expressed in atomic units for momentum will apply to every hydrogenic momentum. Wave function that can be written:

$$F_{nl} = \frac{2^{2l+4} l! \pi}{\left(\frac{z}{na_0}\right)^{\frac{3}{2}} (\hbar)^{\frac{3}{2}}} \left(\frac{n(n-l-1)!}{(n+l)!} \right)^{\frac{1}{2}} \frac{\left(\frac{np}{zp_0}\right)^l}{\left[\left(\frac{np}{zp_0}\right)^2 + 1\right]^{l+2}} C_{n-l-1}^{l+1} \left(\frac{\left(\frac{np}{zp_0}\right)^2 - 1}{\left(\frac{np}{zp_0}\right)^2 + 1} \right) \quad (13)$$

Based on the above equation, if the atomic number for the lithium ion (z) is 2, then the radial momentum wave for the lithium ion can be expressed as follows:

$$F_{nl} = \frac{2^{2l+\frac{5}{2}} \cdot 3^{l+\frac{5}{2}}}{\pi^{\frac{1}{2}}} n^2 l! \left(\frac{(n-l-1)!}{(n+l)!} \right)^{\frac{1}{2}} n^l p^l \times \frac{p_0^{l+\frac{5}{2}}}{[n^2 p^2 + 9p_0^2]^{l+2}} C_{n-l-1}^{l+1} \left(\frac{n^2 p^2 - 9p_0^2}{n^2 p^2 + 9p_0^2} \right)$$

As for the angular equation of momentum formed from two equations, namely the polar equation and azimuth can be expressed as follows:

$$Y_{lm}(\Theta, \Phi) = \left\{ \frac{1}{(2\pi)^{\frac{1}{2}}} e^{\pm im\Phi} \right\} \left\{ \left(\frac{(2l+1)(l-m)!}{2(l+m)!} \right)^{\frac{1}{2}} P_l^m(\cos \Theta) \right\}$$

The flow chart in the calculation of the lithium ion function in the momentum space is as follows:

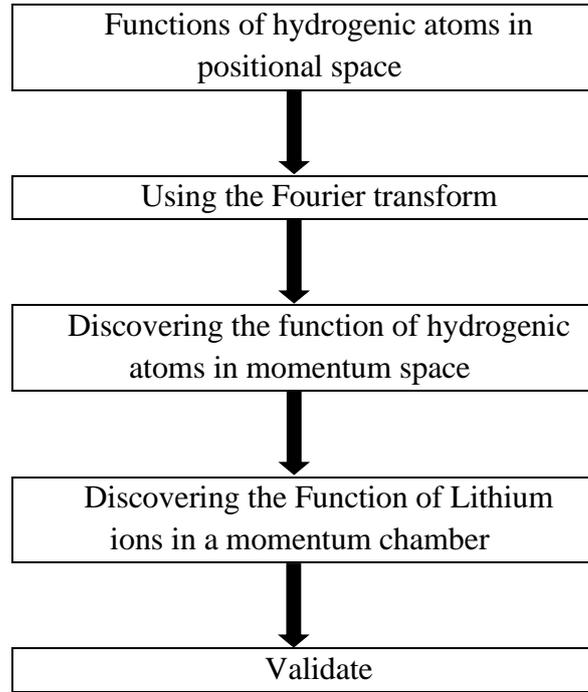


Figure 1. Lithium ion function flowchart in a momentum chamber

The next stage is to obtain results. Results can be obtained from calculations mathematically from the development of theories. The results obtained at this stage will then be validated using the results of previous research. After the results are validated, the next step is to draw conclusions. In this study, there is research that is the main reference material for validation, namely in research conducted by Bransden (1983).

RESULTS AND DISCUSSION

Research on the wave function of Lithium ions (${}_3\text{Li}^{2+}$) on quantum numbers ($n \leq 3$) obtained research results in the form of functions that express wavelengths in momentum space. The Lithium ion (${}_3\text{Li}^{2+}$) is an ion consisting of 3 protons, 3 electrons, and 4 neutrons, so it is hydrogenic because it has a single valence electron. The wave function of Lithium ions (${}_3\text{Li}^{2+}$) in a momentum chamber has two functions, namely the radial wave function and the angular wave function. The radial function is a function that depends on the position (r) of an electron whose shift is parallel to the radius of the Lithium ion. Based on the calculations that have been done, we get the general form of the radial function ($F_{nl}(p)$) is as follows.

$$F_{nl} = \frac{2^{2l+\frac{5}{2}} \cdot 3^{l+\frac{5}{2}}}{\pi^{\frac{1}{2}}} n^2 l! \left(\frac{(n-l-1)!}{(n+l)!} \right)^{\frac{1}{2}} n^l p^l \times \frac{p_0^{l+\frac{5}{2}}}{[n^2 p^2 + 9 p_0^2]^{l+2}} C_{n-l-1}^{l+1} \left(\frac{n^2 p^2 - 9 p_0^2}{n^2 p^2 + 9 p_0^2} \right) \quad (14)$$

Referring to the three general forms of functions above, we get the radial wave function in momentum space, which is presented in table 1 below.

Table 1. Lithium ion radial function ($3Li^{2+}$) in a momentum chamber $n \leq 3$

Skin	n	l	$F_{n,l}(p)$
K	1	0	$\frac{2^{\frac{5}{2}} 3^{\frac{5}{2}}}{\sqrt{\pi}} p_0^{\frac{5}{2}} \frac{1}{(p^2 + 9p_0^2)^2}$
	2	0	$\frac{2^5 3^{\frac{5}{2}}}{\sqrt{\pi}} p_0^{\frac{5}{2}} \frac{(4p^2 - 9p_0^2)}{(4p^2 + 9p_0^2)^3}$
L		1	$\frac{2^7 3^3}{\sqrt{\pi}} p p_0^{\frac{7}{2}} \frac{1}{(4p^2 + 9p_0^2)^3}$
	3	0	$\frac{2^{\frac{5}{2}} 3^8}{\sqrt{\pi}} p_0^{\frac{5}{2}} \frac{(3p^4 - 10p_0^2 p^2 + 3p_0^4)}{(9p^2 + 9p_0^2)^4}$
M		1	$\frac{2^5 3^6}{\sqrt{\pi}} p p_0^{\frac{7}{2}} \frac{(9p^2 - 9p_0^2)}{(9p^2 + 9p_0^2)^4}$
		2	$\frac{2^6 3^8}{\sqrt{5\pi}} p^2 p_0^{\frac{9}{2}} \frac{1}{(9p^2 + 9p_0^2)^4}$

While the angular wave function is an equation consisting of two functions, namely the polar wave function and azimuth wave. The polar wave function depends on the variable θ , where the variable shows the rotational motion of an electron in the X and Y planes. The general form of the polar function is as follows.

$$\Theta_{l,m}(\theta) = (-1)^m \sqrt{\frac{(2l+1)(l-|m|)!}{2(l+|m|)!}} p_l^m \cos\theta \tag{15}$$

The azimuth function relies on a variable φ that shows the rotational motion of electrons around the Z-axis, which is $0 - 2\pi$. The general form of azimuth function is as follows.

$$\Phi_\phi = \sqrt{\frac{1}{2\pi}} e^{\pm im\phi} \tag{16}$$

Based on the polar and azimuth functions above, the general form of the angular wave function can be written as follows.

$$Y_{nlm}(P, \Theta, \Phi) = \left\{ \frac{1}{(2\pi)^{\frac{1}{2}}} e^{\pm im\phi} \right\} \left\{ \left(\frac{(2l+1)(l-m)!}{2(l+m)!} \right)^{\frac{1}{2}} P_l^m(\cos \Theta) \right\} \left\{ -\frac{(i-)^l \pi 2^{2l+4} l!}{(\gamma h)^{\frac{3}{2}}} \left(\frac{n(n-l-1)!}{(n+1)!} \right)^{1/2} \frac{\zeta^l}{(\zeta^2+1)^{l+2}} C_{n-l-1}^{l+1} \left(\frac{\zeta^2-1}{\zeta^2+1} \right) \right\} \tag{17}$$

Referring to the three general forms of functions above, we get the wave function in momentum space, which is presented in table 2 below.

Table 2. Angular function of lithium ion (${}_3\text{Li}^{2+}$) in momentum space $n \leq 3$

Skin	n	l	m	(θ_p)	(ϕ_p)
K	1	0	0	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2\pi}}$
	2	0	0	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2\pi}}$
L	1	0	0	$\sqrt{\frac{3}{2}} \cos\theta$	$\sqrt{\frac{1}{2\pi}}$
		± 1	$\pm \sqrt{\frac{3}{4}} \sin\theta$	$\sqrt{\frac{1}{2\pi}} e^{\pm i\phi}$	
M	3	0	0	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2\pi}}$
		1	0	$\sqrt{\frac{3}{2}} \cos\theta$	$\sqrt{\frac{1}{2\pi}}$
		± 1	$\pm \sqrt{\frac{3}{4}} \sin\theta$	$\sqrt{\frac{1}{2\pi}} e^{\pm i\phi}$	
	2	0	$\sqrt{\frac{5}{8}} (3 \cos^2 \theta - 1)$	$\sqrt{\frac{1}{2\pi}}$	
		± 1	$\pm \sqrt{\frac{15}{4}} \sin\theta \cos\theta$	$\sqrt{\frac{1}{2\pi}} e^{\pm i\phi}$	
		± 2	$\sqrt{\frac{15}{16}} 3 (\sin^2 \theta)$	$\sqrt{\frac{1}{2\pi}} e^{\pm 2i\phi}$	

After obtaining the radial and angular wave functions, the wave function in the momentum space as a whole on the Lithium ion (${}_3\text{Li}^{2+}$) can be determined. As for the general form of the function, as follows.

$$\varphi(p, \theta_p, \phi_p) = F_{n,l}(p) F_{lm}(\theta_p, \phi_p) \tag{18}$$

Referring to the function above, we get the wave function of Lithium ions (${}_3\text{Li}^{2+}$) in the momentum space, which is presented in the following table 3.

Table 3. Lithium ion wave function (${}_3\text{Li}^{2+}$) in a momentum chamber $n \leq 3$

Skin	n	l	m	$\varphi(p, \theta_p, \phi_p)$
K	1	0	0	$\frac{2^{\frac{3}{2}} 3^{\frac{5}{2}}}{\pi} p_0^{\frac{5}{2}} \frac{1}{(p^2 + 9p_0^2)^2}$
	2	0	0	$\frac{2^4 3^{\frac{5}{2}}}{\pi} p_0^{\frac{5}{2}} \frac{(4p^2 - 9p_0^2)}{(4p^2 + 9p_0^2)^3}$
L		1	0	$\frac{2^6 3^{\frac{7}{2}}}{\pi} p p_0^{\frac{7}{2}} \frac{1}{(4p^2 + 9p_0^2)^3} \cos\theta$
		± 1		$\mp \frac{2^{\frac{11}{2}} 3^{\frac{7}{2}}}{\pi} p p_0^{\frac{7}{2}} \frac{1}{(4p^2 + 9p_0^2)^3} \sin\theta e^{\pm i\phi}$
M	3	0	0	$\frac{2^{\frac{3}{2}} 3^8}{\pi} p_0^{\frac{5}{2}} \frac{(3p^4 - 10p_0^2 p^2 + 3p_0^4)}{(9p^2 + 9p_0^2)^4}$
		1	0	$\frac{2^4 3^{\frac{13}{2}}}{\pi} p p_0^{\frac{7}{2}} \frac{(9p^2 - 9p_0^2)}{(9p^2 + 9p_0^2)^4} \cos\theta$
	± 1		$\mp \frac{2^{\frac{7}{2}} 3^{\frac{13}{2}}}{\pi} p p_0^{\frac{7}{2}} \frac{(9p^2 - 9p_0^2)}{(9p^2 + 9p_0^2)^4} \sin\theta e^{\pm i\phi}$	
	2	0		$\frac{2^4 3^{\frac{17}{2}}}{\pi} p^2 p_0^{\frac{9}{2}} \frac{1}{(9p^2 + 9p_0^2)^4} (3 \cos^2 \theta - 1)$
± 1			$\mp \frac{2^{\frac{9}{2}} 3^{\frac{17}{2}}}{\pi} p^2 p_0^{\frac{9}{2}} \frac{1}{(9p^2 + 9p_0^2)^4} \sin\theta \cos\theta e^{\pm i\phi}$	
± 2			$\frac{2^{\frac{7}{2}} 3^{\frac{17}{2}}}{\pi} p^2 p_0^{\frac{9}{2}} \frac{1}{(9p^2 + 9p_0^2)^4} \sin^2 \theta e^{\pm i2\phi}$	

Based on the table above, the main quantum numbers used $n \leq 3$ are $n = 1, 2, 3$. The principal quantum number (n) expresses the energy level of an electron in an ion, where the greater the value (n), the greater the energy of the electron (Fuadah et al., 2018). As for Bohr's theory, the principal quantum number (n) is related to solving problems in radial equations (Damayanti et al., 2020). The application of the main quantum number to the wave function of the Lithium ion in the momentum space can be represented to find the values of probability and expectation. The general equation forms the probability of Lithium ions in momentum space as follows.

$$P(p) = \int_0^\infty p^2 |F_{n,l}(p)|^2 dp \tag{19}$$

(Joachain, 1983)

While the general equation forms the expectation of Lithium ions in the momentum space as follows.

$$P\langle p \rangle = \int_0^\infty p^3 \int_0^\infty p^2 |F_{n,l}(p)|^2 dp$$

(Aruldhas, n.d.) (20)

In quantum numbers the orbital (l) can be used to express the angular velocity of an electron. The greater the value (l), the greater the angular velocity possessed by an electron. Meanwhile, the magnetic quantum number (m) expresses the orbital orientation of an electron. This magnetic quantum number (m) is related to solving problems in the azimuth equation.

In general, the wave function of lithium ions in momentum space is not much different from the wave function of hydrogenic atoms in other momentum spaces. This is shown by the similarity in the angular equations of the two wave functions in the momentum space. However, there is a fundamental difference between the two wave functions in the momentum space, which lies in the value of the radial function. This is because there is a difference in the value (z) or atomic number, where the value (z) on the Lithium atom is 3 while the value (z) on the Hydrogen atom is 1.

CONCLUSION

Based on the results of research that has been done, obtained the wave function of Lithium ions (${}_3\text{Li}^{2+}$) in the momentum space on $n \leq 3$. The wave function consists of radial, angular wave functions (polar and azimuth), and Lithium ion wave functions (${}_3\text{Li}^{2+}$) as a whole. The fundamental difference between the wave function of the Lithium ion (${}_3\text{Li}^{2+}$) in a momentum space and the wave function of a hydrogenic atom in another momentum space is in the value of the radial function caused by the difference in the value of (z) or atomic number.

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